

UNIVERSITY OF WATERLOO
FACULTY OF ENGINEERING
Department of Electrical &
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ECE 204 *Numerical methods*

Approximating the solution to a system of linear algebraic equations

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Approximating solutions to a system of linear equations


Introduction

- In this topic, we will
 - Review what has already been covered:
 - Gaussian elimination with partial pivoting
 - The Jacobi method
 - Describe what will be covered in this topic
 - The Gauss-Seidel method
 - The method of successive over-relaxation
 - Review an implementation of the Jacobi method in MATLAB

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Approximating solutions to a system of linear equations




Looking back

- We have seen how we can use Gaussian elimination with partial pivoting together with backward substitution to approximate the solution to a system of linear equations
 - We say approximate because we are using double-precision floating-point numbers
- We have also seen the iterative Jacobi method
 - This is useful only for strictly diagonally dominant matrices
 - The absolute value of each diagonal entry is greater than the sum of the absolute values of all other entries in that row




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Looking ahead

- We will look at two additional techniques:
 - The Gauss-Seidel method
 - A slight modification of the Jacobi method
 - The method of successive over-relaxation
 - The idea of pushing ahead in the same direction we're already going



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Review of the Jacobi method

- Suppose we are solving $A\mathbf{u} = \mathbf{v}$ and \mathbf{u} is the current approximation:


```


for k = 1:max_iterations
    u_old = u;

    for i = 1:n
        u(i) = v(i);

        for j = [1:i-1, i+1:n]
            u(i) = u(i) - a(i,j)*u_old(j);
        end

        u(i) = u(i)/a(i,i);
    end

    if norm( u - u_old ) < eps_step
        return; // returns 'u'
    end
end
      
```


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Summary

- Following this topic, you now
 - Have reviewed what we have seen so far
 - Understand the next two topics
 - Have reviewed an implementation of the Jacobi method

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References


[1] https://en.wikipedia.org/wiki/Numerical_analysis



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Acknowledgments

None so far.



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Colophon

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The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.



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